There is one peculiarity: The second-order observer we have derived is not minimal order; it always has P additional states (where P is the number of outputs) from the η-filter (9), that is, the dimension is $2N + P$ instead of $2N$. Perhaps this is another perverse byproduct of the unnaturality of this observer. All of this trouble evaporates if $K_1 = 0$, that is, $\dot{\hat{q}} = \hat{\dot{q}}$, but this can rarely be permitted. Therefore, we are forced to accept unnatural second-order observers even in Colorado (see Ref. 5).

VI. Conclusions

We have shown that all linear flexible structures do have second-order observers with arbitrary rates of convergence. However, the most natural definition of such an observer does not work and we are forced to accept an unnatural second-order observer to do the job.

Acknowledgment

We are grateful for the support of the National Renewable Energy Laboratory under Contract XA-0-2-12236-01.

References

[1]Joshi, S. M., "Design of Optimal Second-Order State Estimators," *Journal of Guidance, Control, and Dynamics*, Vol. 14, 1991, pp. 466–468.

[2]Juang, J. N., and Maghami, P. G., "Robust Eigensystem Assignment for State Estimators Using Second-Order Models," *Journal of Guidance, Control, and Dynamics*, Vol. 15, 1992, pp. 920–927.

[3]Hashemipour, H., and Laub, A. J., "Kalman Filtering for Second-Order Models," *Journal of Guidance, Control, and Dynamics*, Vol. 11, 1988, pp. 181–186.

[4]Tseng, Y. W., and Yedavalli, R. K., "Acceleration Based Smart Structure Control Design for Aerospace Applications," *Proceedings of the American Control Conference*, Albuquerque, NM, 1997.

[5]U.S. Supreme Court Decision: Colorado State Constitutional Amendment 2, 1997.

Time-Domain Identification of Low-Order Models for Flexible Structures

Robert J. Bauer*
Dalhousie University,
Halifax, Nova Scotia B3J 1Z1, Canada
and
Peter C. Hughes†
University of Toronto, Toronto, Ontario, M3H 5T6, Canada

I. Introduction

ACCURATE system identification (SI) is often a precursor to modern control system designs that assume that the plant dynamics are precisely known. SI algorithms such as Fast Eigensystem Realization Algorithm (Fast ERA)[1] require Markov parameters to realize discrete-time, linear, time-invariant state-space models. Markov parameters are the discrete-time sequence impulse response measurements from the system and can be obtained using Observer Kalman filter Identification (OKID).[2] To obtain small model prediction errors using methods such as OKID, the size of the identified state-space model must be significantly larger than the expected system size, a process called overspecification. These overspecified models contain not only the principal modes related to the system but also extraneous modes related to measurement errors, nonlinearities, and numerical roundoff errors. The extraneous modes can be eliminated after the identification process through model order reduction because they usually contribute negligibly to the response of the identified system.

The model overspecification and reduction process can impose significant computational resource requirements as well as introduce numerical ill-conditioning problems. To overcome these difficulties methods such as augmented SI[3] have been shown to help reduce the level of overspecification. In the following sections we introduce an SI technique based on cubic smoothing splines that can eliminate the need for model overspecification.

II. Daisy Facility

The Daisy structure, shown in Fig. 1, emulates large, flexible space structures. Daisy consists of a rigid hub, representative of the main body of a satellite, that has three rotational degrees of freedom (roll, pitch, and yaw) aligned with the principal axes. Attached to the hub are 10 slender rigid ribs forming a cone. Each rib is attached at its mass center to the hub via universal joints and is free to rotate about its pivot in two mutually perpendicular directions: in-cone and out-of-cone (by analogy to in-plane and out-of-plane). For SI purposes, daisy is randomly excited by three reaction wheels located in the hub. Rib motion is sensed via an optical deflection sensing system for measuring rib deflections relative to the hub. Attitude encoders measure hub angular displacements.

III. Proposed SI Approach

The proposed SI approach uses successive application of OKID and Fast ERA in combination with a cubic smoothing spline algorithm to yield low-order accurate models. Reaction wheel rates are used as the inputs to OKID and all 23 degrees of freedom are measured. We use $l = 2400$ data samples collected at 10 Hz and select an OKID observer order of $p = 10$. To quantitatively evaluate the performance of a model, the predicted output is compared with the desired output over the experiment time interval as follows:

$$\text{prediction error} = \frac{\|\text{desired} - \text{predicted}\|_2}{\|\text{desired}\|_2} \tag{1}$$

When Fast ERA is applied to OKID's Markov parameters using an observer order of $p = 10$, the resulting model of order $n = 230$ has a prediction error of 0.155. We note that the difference between the impulse response of this identified model and OKID's Markov parameters is on the order of 10^{-4}. Our observation indicates that Fast ERA is successful at identifying a model that matches OKID's Markov parameters; therefore, OKID is responsible for virtually all of the prediction errors observed with the identified model.

The level of overspecification is related to the observer order p used by OKID; therefore, as the observer order increases, the identified model size increases and the model prediction error decreases. Using the Markov parameters for $p = 10$, Fast ERA realizes

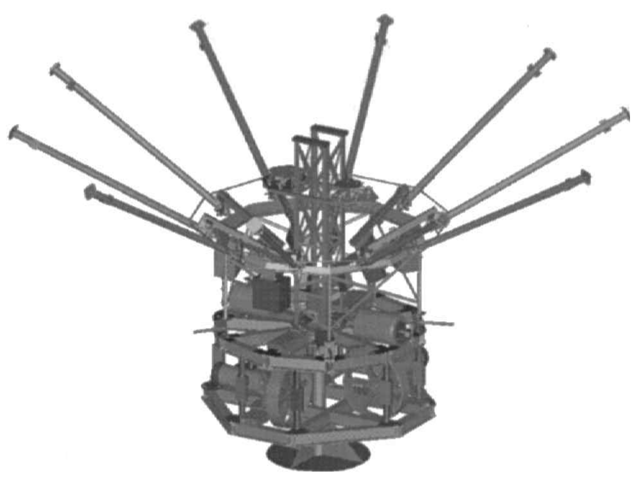

Fig. 1 Daisy facility.

Received 3 March 1998; revision received 1 July 1999; accepted for publication 12 July 1999.

*Assistant Professor, 5269 Morris Street; robert.bauer@dal.ca.

†Professor, Institute for Aerospace Studies, 4925 Dufferin Street; hughesp@ecf.utoronto.ca.

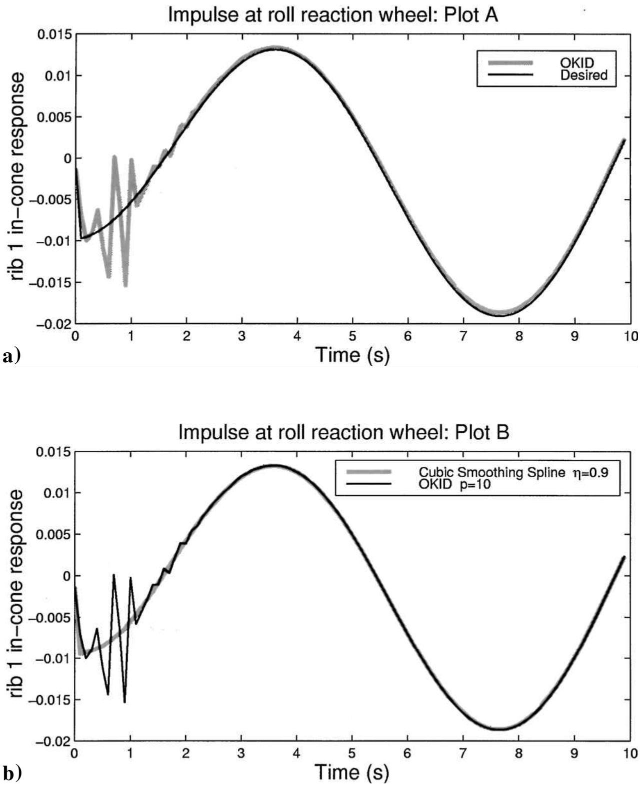

Fig. 2 Sample of cubic smoothing spline curve fit.

a model of order $n = 191$. Balanced model reduction[4] yields a model of order $n = 32$ having an average prediction error of 0.16. Sample in-cone impulse responses of this low-order (desired) model are compared with OKID's system Markov parameters for $p = 10$ in Fig. 2a. It is evident that the high frequencies in the OKID Markov parameters are not required to obtain accurate models. These high frequencies are due to realities in the data such as measurement errors, time-varying parameters, system nonlinearities, and computer roundoff errors, all of which are being modeled with large observer orders.

We propose curve fitting as an approach to removing the high frequencies. Smooth piecewise polynomial functions, called splines, are applied to OKID's Markov parameters. Let the Markov parameters involved in the curve fit span the interval $[a, b]$. Then this interval is subdivided into smaller intervals $[\xi_j, \xi_{j+1}]$ such that $a = \xi_1 < \cdots < \xi_{l+1} = b$, where l is the number of break-points describing the spline. On each interval $[\xi_j, \xi_{j+1}]$ a polynomial of low degree provides a good approximation to the local Markov parameters. The polynomial pieces also blend smoothly so that the resulting patched function has several continuous derivatives. To obtain a spline, we employ the variational approach[5] wherein splines are obtained as the interpolant that minimizes a specific criterion. In particular, given the data (x_i, y_i) with $x_i \in [a, b] \, \forall i$, the cubic smoothing spline minimizes the cost function C

$$C = \eta \sum_{i=a}^{b} [y_i - f(x_i)]^2 + (1 - \eta) \int_a^b \left(\frac{\mathrm{d}^2 f(t)}{\mathrm{d}t^2} \right)^2 \mathrm{d}t \qquad (2)$$

over all functions f with two derivatives, where η is called the smoothing parameter. The smoothing spline is a cubic spline with a breakpoint at every data point. Smoothing is achieved by selecting $\eta \in [0, 1]$ so that the spline contains as much of the information and as little of the unwanted high frequencies in the data as possible. Selecting $\eta = 1$ yields a least-squares straight line fit whereas $\eta = 0$ gives the natural cubic spline interpolant.

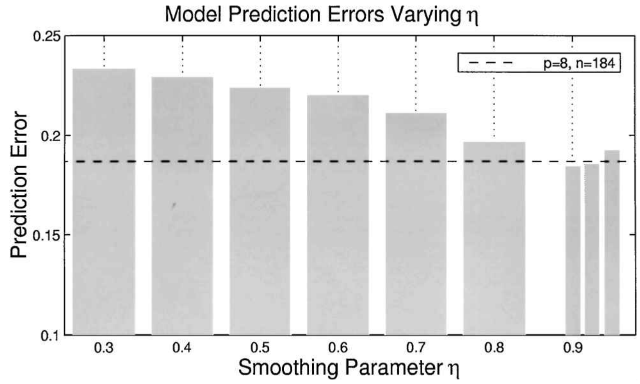

Fig. 3 Minimizing prediction error by varying η.

IV. Experimental Results

Sample results of a cubic smoothing spline fit to OKID's Markov parameters for $p = 10$ are shown in Fig. 2b. The high frequencies at the beginning of each impulse response are effectively eliminated whereas the curve fit still follows the rest of the response. Using 1000 system Markov parameters, the first 500 of which are curve fitted using a smoothing parameter of $\eta = 0.9$, Fast ERA can identify a model of order $n = 46$ directly. The prediction error for this low-order model is 0.185. The resulting prediction errors for various values of η are shown in Fig. 3. As η increases from 0.3 to 0.9, the resulting model prediction error decreases. A minimum prediction error occurs near $\eta = 0.9$, and after this point the prediction error begins to increase. This increase is expected because beyond $\eta = 0.9$ there is very little smoothing of the data and a model of order $n = 46$ is not sufficient to capture the high frequencies. To get equivalent results using standard SI techniques, Fig. 3 demonstrates that a model of order $n = 184$ would have to be identified and then reduced to $n = 46$. Computational requirements are reduced by 43% and numerical ill-conditioning problems associated with model overspecification and reduction are avoided.

V. Conclusion

A new approach to system identification has been proposed that applies cubic smoothing splines to the system Markov parameters calculated by OKID. Applying this smoothing procedure early in the identification process allows models that are both low order and accurate to be directly identified. Experiments with daisy, a flexible spacecraft emulator, demonstrate significant computational savings and numerical robustness by avoiding the model overspecification and reduction process.

Acknowledgments

This research was sponsored by the Natural Sciences and Engineering Research Council of Canada (NSERC). The authors wish to thank Tony Hong of UTIAS for helping with the Daisy experiments.

References

[1] Peterson, L. D., "Efficient Computation of the Eigensystem Realization Algorithm," *Proceedings of the 10th International Modal Analysis Conference*, Vol. 2, 1992, pp. 1122–1131.

[2] Juang, J., Phan, M., Horta, L. G., and Longman, R. W., "Identification of Observer/Kalman Filter Markov Parameters: Theory and Experiments," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 2, 1993, pp. 320–329.

[3] Hong, T., Bauer, R. J., Hughes, P. C., Carroll, K. A., and Vukovich, G., "Laboratory Results on 'Augmented' System Identification of Flexible Spacecraft with Nonlinearities," *Canadian Aeronautics and Space Journal*, Vol. 43, No. 3, 1997, pp. 169–175.

[4] Moore, B. C., "Principal Component Analysis in Linear Systems: Controllability, Observability and Model Reduction," *IEEE Transactions on Automatic Control*, Vol. 26, No. 1, 1981, pp. 17–32.

[5] Späth, H., *Spline Algorithms for Curves and Surfaces*, Utilitas Mathematica Publ., Winnipeg, MB, Canada, 1974, pp. 106–123.